Improving and Using R-X Noise Bridges

Here's how to improve your noise bridge's measurement accuracy and capability, and some ways to put your modified noise bridge to work.

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The R-X noise bridge is one of the most useful pieces of test equipment available to radio amateurs. Noise bridges can be used to measure antenna impedances, coaxial-cable losses and characteristic impedances, and impedances of components at different operating frequencies. You can do all of this with commercial R-X noise bridges that cost less than $60.

Impedance measurements are very important in design and construction of antennas. Knowing the impedance of an antenna can allow you to tune it for a more optimal match than you could with only an SWR bridge. Knowing how an antenna's impedance varies as a function of frequency can allow you to design a matching network that operates over the desired frequency range.

Unfortunately, the noise bridge has not lived up to its potential. Many past and current designs yield inaccurate measurements. Furthermore, I've never seen a good description of a procedure for accurately calibrating the reactance scale of a noise bridge. Finally, published articles on noise-bridge use have stipulated that antenna-impedance measurements must be made either at the antenna, or at the end of a coaxial cable that is a multiple of a half wavelength at the antenna's operating frequency. Cutting such cables is difficult to do, and even perfect-length cables can introduce significant errors into the measurement process. At best, using such cables yields accurate results at only one frequency.

In this article, I'll describe a few simple changes that can be made to existing noise-bridge designs to significantly increase their accuracy. I'll also describe a method for calibrating noise-bridge resistance and reactance scales. In addition, I'll show how to use a noise bridge to measure cable loss, characteristic impedance and electrical length, and to measure the impedance of an antenna—even if it's 100 feet in the air, at the end of an arbitrary length of coaxial cable.

I modified and calibrated my noise bridge, a Palomar Engineers unit, using the procedures in this article.

Noise-Bridge Design

The block diagram of a noise bridge is shown in Fig 1. It consists of a noise source that may or may not be modulated, an amplifier, and bridge. An unknown impedance is measured by connecting it to the UNKNOWN connector. A receiver attached at the RECEIVER connector is used to detect bridge balance.

The bridge is balanced when the reactance in the bridge's upper arm equals that in the lower arm. Bridge balance is achieved by adjusting the variable resistor (R_v) and capacitor (C_v) until a null is detected in the receiver. Impedance measurements are made at the frequency to which the receiver is tuned. The receiver should be operated in its AM-detection mode, but adequate results can be achieved in CW or SSB modes if AM is not available.

Throughout this article, I'll use the series impedance representation. Impedance is a complex quantity, of which the resistance is the real part and the reactance is the imaginary part. Both of these are given in ohms; *j* denotes the imaginary part. The impedance of a circuit that has a resistance of R and a reactance of X is represented as

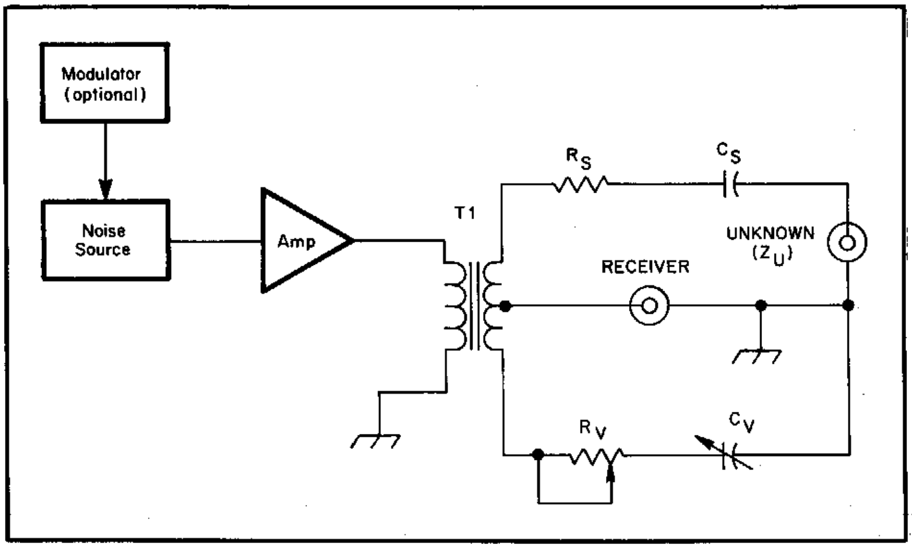


Fig 1—Block diagram of an R-X noise bridge. R_v and C_v are the variable resistor and capacitor used to balance the bridge. Z_u is the unknown impedance, which has resistive (R_u) and reactive (X_u) components. R_s and C_s are in series with the unknown impedance.

$$Z = R + jX \quad \text{(Eq 1)}$$

The bridge is in balance when

$$R_u = R_v - R_s \quad \text{(Eq 2)}$$

and

$$X_u = X_v - X_s \quad \text{(Eq 3)}$$

where

R_u and X_u represent the resistive and reactive parts, respectively, of the unknown impedance

X_v and X_s represent the reactances of the bridge capacitors.

More information about impedances is presented in *The ARRL Handbook*[1] and *The ARRL Extra Class License Manual*.[2]

Checking Noise-Bridge Accuracy

To check the accuracy of a noise bridge, you must use good calibration loads. Those described here consist of a short-circuit load, a 50-Ω load, a 180-Ω load, and a variable-resistance load. The short-circuit and fixed-resistance loads are used to check the accuracy of the noise bridge; the variable-resistance load is used in measuring coaxial-cable loss.

Construction details of each of these loads are shown in Fig 2. Each load is constructed inside the body of a UHF plug (PL-259) connector. When building these loads, keep leads as short as possible to minimize parasitic effects. The resistors must be noninductive (*not* wirewound). Quarter-watt, carbon-composition resistors should work fine. The potentiometer in the variable-resistance load is a miniature PC-mount unit with a maximum resistance of 100 Ω or less. The potentiometer's wiper and one of the end leads are connected to the center pin of the PL-259; the other lead is connected to ground.

The first step in noise-bridge calibration is making sure that the bridge's measurements do not vary with frequency. Connect a receiver to the bridge's RECEIVER connector, and hook the short-circuit load to the UNKNOWN connector. Tune the receiver to the lowest operating frequency of the bridge, and null the bridge by adjusting the variable resistor and capacitor until a dip in signal strength occurs in the receiver. In this state, a perfectly calibrated bridge will indicate zero ohms on both the resistance and reactance scales.

When a null is reached, increase the receiver frequency by a few megahertz and repeat the measurement. The resistance and reactance readings shouldn't change. Repeat this procedure until you reach the bridge's highest operating frequency.

Check the bridge at a frequency a few megahertz above the bridge's minimum frequency with the 50- and 180-Ω loads. With these loads, the bridge should indicate the appropriate resistance (50 or 180 Ω) and zero reactance. Resistive loads may show some negative reactance at higher frequencies; this results from the capacitance (about 5 pF) of the connector. At 30 MHz, these reactances are about −2 Ω for the 50-Ω load and −30 Ω for the 180-Ω load.

Reactance readings should remain constant when frequency is varied with the short-circuit load, as they should at low frequencies with the fixed-resistance loads. With the short-circuit load, resistance readings should remain constant as frequency is varied.

If the noise bridge passes these tests, the design is good, and you can skip the section on improvements. My experience with several commercial noise bridges has shown me that most are deficient; measured resistances or reactances vary with frequency—not a good thing. The modifications described in the next section allow you to vastly improve a suboptimal noise bridge.

Noise-Bridge Improvements

The improved bridge is shown in Fig 3. I applied these modifications to my noise bridge; these changes should work in other noise bridges as well.

The main cause of frequency variation of the bridge null is the design of the transformer (T1 in Fig 1). The designs that I've seen use a trifilar winding on a ferrite or powdered-iron toroid core. This design introduces phase shift across the transformer secondaries at the higher frequencies, causing the reactance at bridge balance to vary with frequency. Some transformers also have insufficient permeability in the toroidal core; this causes resistance readings to vary at lower frequencies.

Both of these problems can be cured by using a ferrite binocular core wound as shown in Fig 4. This binocular transformer directly replaces the toroidal transformer in the bridge. This design eliminates phase

[1]Notes appear on p 32.

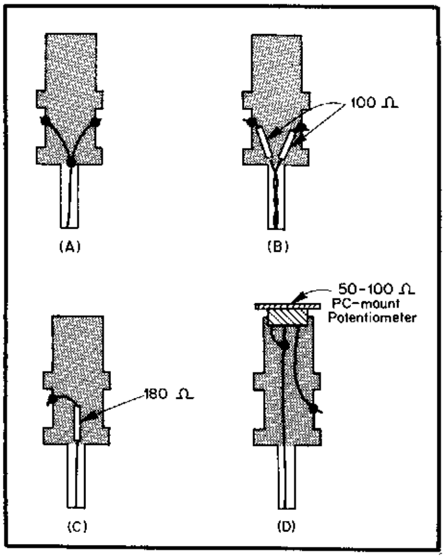


Fig 2—Construction details of the resistive loads used to check and calibrate a noise bridge. Each of the loads is constructed inside a PL-259 connector. (Views shown are cross-sections of the PL-259 bodies only; the sleeves are not shown.) Leads should be kept as short as possible to minimize parasitic inductance. (A) is a short circuit; (B) depicts a 50-Ω load; (C) is a 180-Ω load; (D) shows a variable-resistance load used to determine the loss in a coaxial cable.

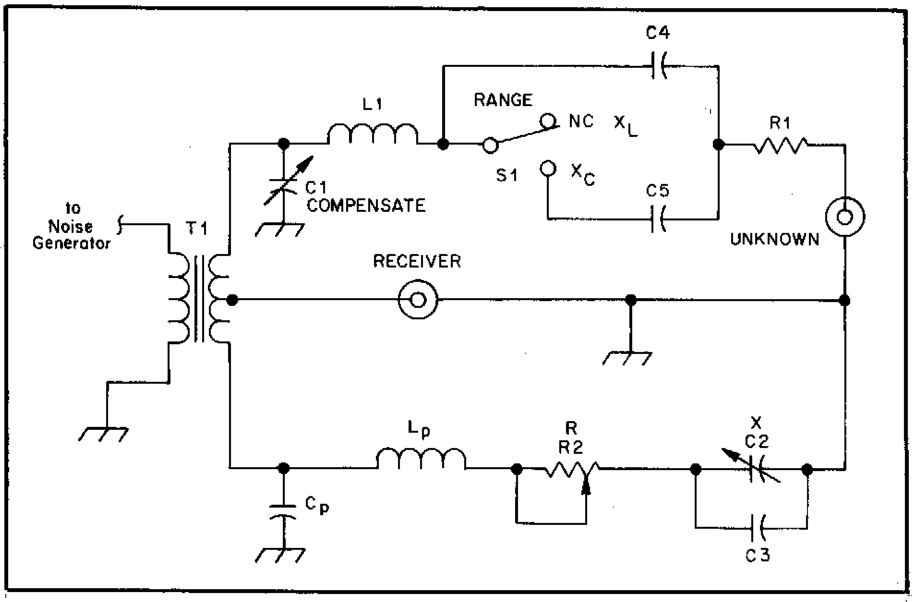


Fig 3—Detailed circuit diagram of the improved noise bridge. This circuit was used to modify my noise bridge. The existing variable resistor (R2) and variable capacitor (C2) are used in the modified circuit. The parasitic capacitance and inductance due primarily to R2 are shown as C_p and L_p in the circuit diagram.

Parts List

C1—2- to 8-pF variable; see text.
C2—15- to 150-pF variable.
C3—20-pF mica.
C4—47-pF mica.
C5—82-pF mica.
L1—Wire loop; see text.
R1—10-Ω, ¼ W.
R2—250-Ω, noninductive potentiometer.
S1—SPDT toggle, Radio Shack® 275-625.
T1—Each winding consists of 3 turns of insulated, solid wire on an Amidon BLN-43-2402 ferrite core. See Fig 4 for winding detail.

shift, and the ferrite core has sufficient permeability to eliminate low-frequency resistance shift.

Stray Capacitance

After I installed the binocular transformer in my noise bridge, I discovered a second problem that was due to stray capacitance. Stray capacitance on the variable-resistor side of the bridge tends to be higher than that on the unknown side, primarily because the parasitic capacitance in the variable resistor, R_v is comparatively high. This capacitance is shown by C_p in Fig 3.

The effect of C_p is most easily detected using the 180-Ω load. Connect this load to the UNKNOWN connector, tune the receiver to the lowest operating frequency of the bridge, and null the bridge. Use an ohmmeter to measure the dc resistance of the bridge's variable resistor. If this resistance is greater than the dc resistance of the 180-Ω load plus series resistor R_s, the stray capacitance is greater on the variable-resistor side of the bridge. The magnitude of the stray capacitance can be calculated by

$$C_p = C_s\left(\sqrt{\frac{R_v}{R_L + R_s}} - 1\right) \qquad \text{(Eq 4)}$$

where
R_L = load resistance (in this case, 180 Ω)
R_v = resistance of the variable resistor
C_s = series capacitance

This stray capacitance can be compensated for by placing a variable capacitor (C1 in Fig 3) in the unknown side of the bridge. If the required compensating capacitance is only a few picofarads, you can use a gimmick capacitor (made by twisting two short pieces of insulated, solid wire together). To adjust the value of this compensating capacitor, set the dc resistance of the noise-bridge variable resistor, R_v, equal to the dc resistance of the 180-Ω load plus the series resistor. Then, adjust the compensating capacitor (by trimming its length) and the bridge's variable capacitor until a null is obtained. Make this adjustment at the bridge's lowest operating frequency. A capacitance of 7.5 pF was required to balance my bridge.

Stray Inductance

A second undesirable parasitic effect results from stray inductance in the variable resistor, shown as L_p in Fig 3. You can detect this stray inductance by placing the short-circuit load on the UNKNOWN connector and measuring the reactance at the lowest and highest operating frequencies; the reactance indicated should be the same at both frequencies. If the reactance reading decreases as frequency is increased, there is parasitic inductance in the variable-resistor leg, and a compensating inductance needs to be placed in the unknown leg. If reactance increases with frequency, the extra inductance is in the unknown leg, and the compensating inductance must be placed in

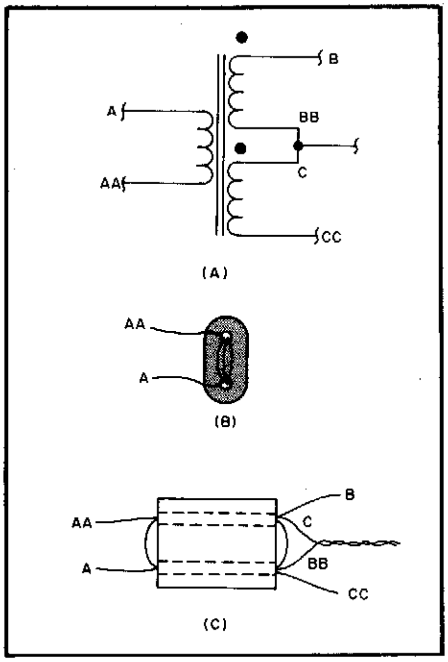


Fig 4—Detail of noise-bridge-transformer construction. At A, the circuit diagram of the transformer. B and C are the end and side views, respectively. The transformer is constructed on an Amidon BLN 43-2402 ferrite core. Each winding is three turns of no. 30 enameled wire. One turn is equal to the wire passing once through both holes in the ferrite core. The primary winding starts on one side of the transformer, and the secondary winding starts on the other side.

the variable-resistance leg.

The parasitic inductance, if present, should be only a few tens of nanohenries. This represents a few ohms of inductive reactance at 30 MHz.

The stray inductance is compensated by placing a single-turn coil, made from a 1- to 2-inch length of solid wire, in the appropriate leg of the bridge. Adjust the size of this coil until the reactance reading remains constant from the lowest to the highest operating frequency. A one-turn loop made from a 1¼-inch length of no. 26 wire, placed in the unknown leg, was required to balance my bridge.

Reactance-Measurement Range

The reactance range of a noise bridge is dependent on several factors, including operating frequency, value of the series capacitor (C4/C5 in Fig 3), and the range of the variable capacitor (C_v in Fig 1). For example, at 10 MHz, the reactance range of my unmodified bridge is from −800 Ω to 130 Ω. This is clearly biased toward the capacitance side. I've measured similar reactance ranges on other designs.[3]

You can determine the range of reactances that your noise bridge should cover from the magnitude of the SWR that you'll need to measure with the bridge. I wanted my bridge to be capable of measuring an SWR of 5:1, referenced to 50 Ω, at 30 MHz. To do this, the bridge must cover a resistance range of 5 to 250 Ω and a reactance range of −120 to 120 Ω. At 10 MHz, this is equivalent to covering reactances from −360 to 360 Ω. This measurement range is nearly achieved in the design shown in Fig 3. (The resistance and reactance ranges, after modification, are 0 to 220 Ω and −400 to 230 Ω, respectively, at 10 MHz. Measurement error is estimated to be ±5 Ω [±10% of impedance].)

A 20-pF capacitor in parallel with C2 limits C2's range. I added a RANGE switch to select reactance measurements weighted toward either capacitance or inductance. The zero-reactance point occurs when C2 is either nearly fully meshed or fully unmeshed. The RANGE switch nearly doubles the resolution of the reactance readings.

I mounted the switch on the back panel of my bridge, near the UNKNOWN connector. I also added a larger front panel and larger knobs to the bridge to achieve better resolution for the resistance and reactance readings. A template for the front panel is available from me for an SASE.

Calibration

Good calibration accuracy is necessary for accurate noise-bridge measurements. Calibration of the resistance scale is straightforward. To do this, tune the receiver to a frequency near the center of the bridge's range; generally, this is around 10 MHz. Attach the short-circuit load to the UNKNOWN connector and null the bridge. This is the zero-resistance point, and should be marked on the front-panel resistance scale. The rest of the resistance range is calibrated, using an accurate ohmmeter, by measuring the increase in resistance as variable resistor R2 is adjusted, and then marking these increments on the front panel; I marked the scale at 10-Ω increments.

How you perform the reactance calibration depends on how you want to read reactances on the bridge. Most of the published calibration methods provide reactance readings in terms of capacitance. My method, however, provides calibration of the reactance scale in *ohms*, at a reference frequency of 10 MHz. The advantage of this method is that it gives an answer in units that are most relevant in impedance measurement. The disadvantage of this method is that the readings must be scaled if the measurement frequency is something other than 10 MHz. The equation for performing this scaling is:

$$X_u(f) = X_{u_{10}} \frac{10}{f} \qquad \text{(Eq 5)}$$

where
f = frequency in MHz
$X_{u_{10}}$ = unknown reactance at 10 MHz

My reactance-calibration method requires only a shorted piece of coaxial cable to serve as a reactance source. (The reactance of a shorted, low-loss coaxial cable is dependent only on the cable length, the measurement frequency and the cable's characteristic impedance.) To calibrate my bridge, I used Radio Shack® RG-8M because it is readily available, has relatively low loss, and has an almost purely resistive characteristic impedance.

Follow these steps to calibrate the reactance scale:

1) Cut a length of coaxial cable that is slightly longer than ¼ λ (246 $v_p \div f_{MHz}$; about 20 feet for RG-8M). Attach a PL-259 connector to one end of the cable; leave the other end open-circuited. This cable will be used as the calibration standard for the reactance scale.

2) Connect the short-circuit load to the noise bridge's UNKNOWN connector and set the receiver frequency to 10 MHz. Adjust the noise bridge for a null. Do not adjust the reactance control after the null is found.

3) Connect the calibration cable to the bridge's UNKNOWN terminal. Adjusting only the variable resistor and the receiver frequency, null the bridge. The receiver frequency should be less than 10 MHz; if it is above 10 MHz, the cable is too short, and you'll need to prepare a new one.

4) Gradually cut short lengths from the end of the coaxial cable until you obtain a null at 10 MHz by adjusting only the resistance control. Then connect the cable's center and shield conductors at the far end with a short length of braided cable. Verify that the bridge nulls with zero reactance at 20 MHz. My calibration cable turned out to be 18 feet, 2 inches long.

5) The reactance scale is now ready for calibration. The reactance of the coaxial cable (normalized to 10 MHz) can be calculated from:

$$X_{i_{10}} = R_0 \frac{f}{10} \tan\left(2\pi \frac{f}{40}\right) \qquad \text{(Eq 6)}$$

where

$X_{i_{10}}$ = cable reactance at 10 MHz
R_0 = characteristic resistance of the coaxial cable
f = frequency in MHz
R_0 is 52.5 Ω for Radio Shack RG-8M.
(Note: all trig functions are in radians.)

The results of Eq 6 have less than 5% error for reactances less than 500 Ω, as long as the loss in the test cable is less than 0.2 dB. This error becomes significantly less at lower reactances (2% error at 300 Ω for a 0.2-dB-loss cable). The loss in 18 feet of RG-8M is 0.13 dB at 10 MHz. Data for using Radio Shack RG-8M is given in Table 1.

6) Tune the receiver to the appropriate frequency for the desired reactance (given in Table 1, or found using Eq 6). Adjust the resistance and reactance controls to null the bridge. Mark the reactance reading on the front panel. Repeat this process until all desired reactance values have been marked. The resistance values needed to null the bridge during this calibration procedure may be significant (more than 100 Ω) at the higher reactances.

Table 1
Noise-Bridge Calibration Data: Coaxial-Cable Method
This data is for Radio Shack RG-8M cable cut to exactly ¼ λ at 10 MHz; the reactances (calculated) correspond to this frequency.

X_i (10 MHz)	Frequency (MHz)	X_i (10 MHz)	Frequency (MHz)
10	3.318	−10	19.376
20	4.484	−20	18.722
30	5.262	−30	18.048
40	5.838	−40	17.368
50	6.286	−50	16.701
60	6.647	−60	16.063
70	6.943	−70	15.472
80	7.192	−80	14.936
90	7.404	−90	14.461
100	7.587	−100	14.045
110	7.746	−110	13.683
120	7.885	−120	13.369
130	8.009	−130	13.097
140	8.119	−140	12.861
150	8.217	−150	12.654
160	8.306	−160	12.473
170	8.387	−170	12.313
180	8.460	−180	12.172
190	8.527	−190	12.045
200	8.588	−200	11.932
210	8.645	−210	11.831
220	8.697	−220	11.739
230	8.746	−230	11.655
240	8.791	−240	11.579
250	8.832	−250	11.510
260	8.872	−260	11.446
270	8.908	−270	11.387
280	8.942	−280	11.333
290	8.975	−290	11.283
300	9.005	−300	11.236
350	9.133	−350	11.045
400	9.232	−400	10.905
450	9.311	−450	10.798
500	9.375	−500	10.713

This calibration method is much more accurate than using fixed capacitors across the UNKNOWN connector. You can calibrate a noise bridge in less than an hour using this method.

Measuring Coaxial-Cable Parameters with a Noise Bridge

Coaxial cables have a number of properties that affect the transmission of signals through them. Generally, radio amateurs are mainly concerned with cable attenuation and characteristic impedance. However, if you plan to use a noise bridge to make antenna-impedance measurements, you need to accurately determine not just cable impedance and attenuation, but also electrical length. Fortunately, all of these parameters are easy to measure with a noise bridge that's been improved as discussed earlier.

The first parameter that you need to measure is the cable's electrical length. There are a number of ways of expressing the electrical length of a cable, the most common of which is cable length in degrees at a given frequency. We'll express cable length as the frequency at which a cable is one wavelength long. This length will be represented by f_λ. Follow these steps to determine f_λ for a coaxial cable:

1) Tune the receiver to the frequency range of interest. (f_λ varies slightly with frequency, so tune as close as possible to the frequency of interest.) Attach the short-circuit load to the noise bridge's UNKNOWN connector and null the bridge.

2) Disconnect the far end of the coaxial cable from its load (the antenna) and connect the short-circuit load in place of this load. Connect the near end of the cable to the bridge's UNKNOWN connector.

3) Adjust the receiver frequency and the noise-bridge resistance control for a null. *Do not change the noise bridge's reactance-control setting during this procedure.* Note the frequency at which the null is found; call this f_n. The noise-bridge resistance at the null should be relatively small (less than 20 Ω).

4) Tune the receiver upward in frequency until the next null is found. Adjust the resistance control, if necessary, to improve the null; *but do not adjust the reactance control.* Note the frequency at which this second null is found; this is (f_{n+2}).

5) We can now find the value of n and the electrical length of the cable. (Round n to the nearest even integer after calculation.)

$$n = \frac{2 f_n}{f_{n+2} - f_n} \qquad \text{(Eq 7)}$$

$$f_\lambda = \frac{4 f_n}{n} \qquad \text{(Eq 8)}$$

This procedure can also be followed using an open circuit as the termination. However, the end effects on the open-circuit PL-259 increase the effective length of the coaxial cable; this decreases its f_λ. If you use an open-circuit cable following this procedure, n will be an odd number, *not* an even number.

The characteristic impedance of the coaxial cable is found by measuring its input impedance at two frequencies separated by $f_\lambda/4$. This must be done when the cable is terminated in a resistive load. Characteristic impedance changes slowly as a function of frequency, so this measurement must be done near the frequency of interest. The measurement procedure is as follows:

1) Place the 50-Ω load on the far end of the coaxial cable and connect the near end to the UNKNOWN connector of the noise bridge. (Measurement error is minimized when the load resistance is close to the characteristic impedance of the cable. This is the reason for using the 50-Ω load.)

2) Tune the receiver approximately $f_\lambda/8$ below the frequency of interest. Adjust the bridge resistance and reactance controls to obtain a null, and note their readings as R(f) and X(f). Remember, the reactance

measured on the front panel must be scaled to the measurement frequency.

3) Increase the receiver frequency by exactly $f_\lambda/4$. Again, null the bridge and note the readings as $R(f + f_\lambda/4)$ and $X(f + f_\lambda/4)$.

4) Calculate the characteristic impedance of the coaxial cable using Eqs 9 through 14. A scientific calculator is helpful for this.

$$R = R(f) \times R(f + f_\lambda/4) - X(f) \times X(f+f_\lambda/4) \quad \text{(Eq 9)}$$

$$X = R(f) \times X(f + f_\lambda/4) + X(f) \times R(f + f_\lambda/4) \quad \text{(Eq 10)}$$

$$Z = \sqrt{R^2 + X^2} \quad \text{(Eq 11)}$$

$$R_0 = \sqrt{Z} \cos\left(\frac{1}{2} \tan^{-1}\left[\frac{X}{R}\right]\right) \quad \text{(Eq 12)}$$

$$X_0 = \sqrt{Z} \sin\left(\frac{1}{2} \tan^{-1}\left[\frac{X}{R}\right]\right) \quad \text{(Eq 13)}$$

$$Z_0 = R_0 + j\,X_0 \quad \text{(Eq 14)}$$

From taking measurements on a number of types of coaxial cable, I found that nominal 50-Ω cables have characteristic resistances between 45 and 60 Ω, and characteristic reactances between −2 and −10 Ω.

Cable loss can be measured once the cable's electrical length and characteristic resistance are known. The following procedure allows you to measure the cable loss every $f_\lambda/4$ in frequency. Loss between measurement points can be interpolated with reasonable accuracy. This procedure employs a resistor-substitution method, and provides much better measurement accuracy than that achieved by directly reading the resistance from the noise-bridge scale.

1) Determine the approximate frequency at which you want to make the loss measurement by using

$$f = n \times f_\lambda/4 \quad \text{(Eq 15)}$$

where n is any positive integer.

2) If n is odd, leave the far end of the coaxial cable open circuited; if it is even, connect the short-circuit load to the far end of the cable. Attach the near end of the cable to the UNKNOWN connector on the noise bridge.

3) Set the noise bridge to zero reactance. Tune the receiver frequency and the noise bridge resistance to find the null.

4) Connect the variable-resistance load to the UNKNOWN terminal on the noise bridge. Without changing the resistance setting on the bridge, adjust the load resistor and the bridge reactance to obtain a null.

5) Remove the variable-resistance load from the bridge's UNKNOWN terminal and measure the load's resistance using an ohmmeter that's accurate at low resistance levels. Refer to this resistance as R_i.

6) Calculate the cable loss in decibels using

$$\alpha\ell = 8.69 \frac{R_i}{R_0} \quad \text{(Eq 16)}$$

As an example of this method, let's calculate the parameters for a 74-foot length of Columbia 1188 (an RG-58-equivalent cable). We'll make these calculations near the 10-meter band.

With a short-circuit load on the far end of the cable, we measure nulls at 24.412 and 29.353 MHz. This corresponds to an n of 10, an f_λ of 9.765 MHz at 24.412 MHz, and an f_λ of 9.784 MHz at 29.353 MHz. With a 50-Ω load on the far end of the cable, we then make the following resistance and reactance measurements, centered at 28 MHz:

$f = 26.777$ MHz
$R(f) = 64\ \Omega$
$X(f) = -22\ \Omega$
$f + f_\lambda/4 = 29.223$ MHz
$R(f + f_\lambda/4) = 50\ \Omega$
$X(f + f_\lambda/4) = -24\ \Omega$

This corresponds to a characteristic impedance of:

$R_0 = 56.6\ \Omega$
$X_0 = -8.3\ \Omega$
$Z_0 = 56.6 - j8.3\ \Omega$

The input resistance of the cable is 12.1 Ω with a short-circuit load on the far end of the cable at 29.353 MHz; this corresponds to a loss of 1.85 dB.

Using a Noise Bridge to Measure the Impedance of an Antenna

The impedance at the end of a transmission line can be easily measured using a noise bridge. What we really want to measure, though, is the impedance of the antenna—that is, the impedance of the load at the far end of the line. There are several ways to handle this.

1) Measurements can be made with the noise bridge at the antenna. This is usually not practical because the antenna must be in its final position for the measurement to be accurate. Even if it can be done, making such a measurement is certainly not very convenient.

2) Measurements can be made at the end of a coaxial cable—if the cable length is an exact integer multiple of ½ λ. This effectively restricts measurements to a single frequency. Measuring the impedance of an antenna across an entire amateur band using this method results in significant errors.

3) Measurements can be corrected using a Smith chart, as described in *The ARRL Antenna Book*.[4] This graphical method can result in reasonable estimates of antenna impedance—as long as the SWR is not too high and the cable is not too lossy. However, it doesn't compensate for the complex impedance characteristics of real-world coaxial cables. Also, compensation for cable loss can be tricky to apply. These problems, too, can lead to significant errors.

4) Lastly, measurements can be corrected using the transmission-line equation, as discussed in the Appendix. The transmission-line equation can be solved using only a scientific calculator, but this is rather tedious to do if more than a few data points are taken. A better method is to use a programmable calculator or personal computer to perform the calculations. (I have listings for a BASIC program and an HP-41C calculator program to perform these calculations. These listings are available from me for an SASE.) I feel that this is the best method for calculating antenna impedances from measured parameters. But it has the disadvantage of requiring that you measure some of the antenna's feed-line characteristics beforehand—measurements for which you'll need to have access to both ends of the feed line.

The procedure for determining antenna impedance is to first measure the electrical length, characteristic impedance, and attenuation of the coaxial cable connected to the antenna. After making these measurements, connect the antenna to the coaxial cable and measure the input impedance of the cable at a number of frequencies in the antenna's operating-frequency band. Then, use these measurements in the transmission-line equation to determine the actual antenna impedance at each frequency.

Table 2 and and Fig 5 give an example of such a calculation. The antenna used for this example is a 10-meter inverted V about 30 feet above the ground. The legs of the antenna are separated by a 120° angle. Each leg is exactly 8 feet long, and the antenna is made of no. 14 wire. The feed line is the 74-foot length of Columbia 1188 characterized earlier.

See Fig 5A. From this plot of impedance measurements, it is very difficult to determine anything about the antenna. Resistance and reactance vary substantially over this frequency range, and the antenna appears to be resonant at 27.7, 29.0 and 29.8 MHz.

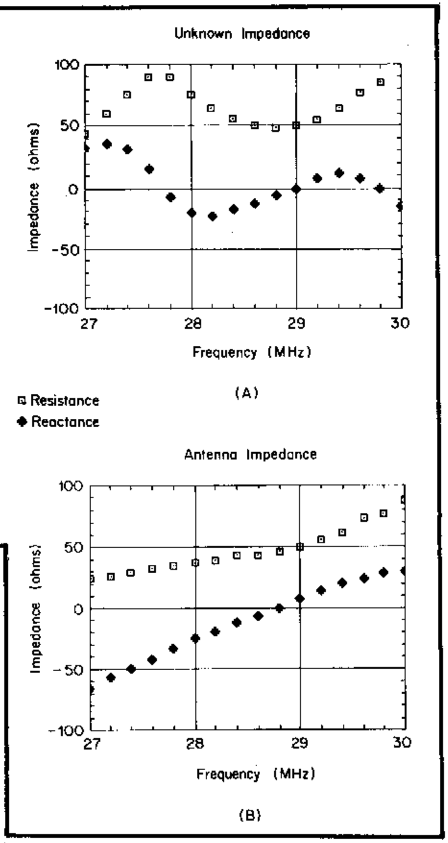
The plot in Fig 5B shows the true antenna impedance. This plot has been corrected for the effects of the cable using the transmission-line equation. This plot shows that the antenna's true resistance and reactance both increase smoothly with frequency. The antenna is resonant at 28.8 MHz, with a radiation resistance at resonance of 47 Ω. This is just about what you'd expect from an inverted V.

When doing these conversions, you must be careful not to make measurement errors. Such errors introduce more errors into the corrected data. This problem is most significant when the transmission line is approximately an odd multiple of a quarter wavelength long and the line SWR and/or attenuation is high. Measurement errors are probably present if small changes in the input impedance or transmission-line characteristics appear as large changes in antenna

Table 2
Impedance Data for Inverted-V Antenna

Freq (MHz)	R_u (ohms)	X_u @ 10 MHz (ohms)	X_u (ohms)	R_L (ohms)	X_L (ohms)
27.0	44	85	31.5	24	−65
27.2	60	95	34.9	26	−56
27.4	75	85	31.0	30	−51
27.6	90	40	14.5	32	−42
27.8	90	−20	−7.2	35	−34
28.0	75	−58	−20.7	38	−24
28.2	65	−65	−23.0	40	−19
28.4	56	−52	−18.3	44	−12
28.6	50	−40	−14.0	44	−6
28.8	48	−20	−6.9	47	1
29.0	50	0	0.0	52	8
29.2	55	20	6.8	57	15
29.4	64	30	10.2	63	21
29.6	78	20	6.8	75	26
29.8	85	0	0	78	30
30.0	90	−50	−16.7	89	33

Fig 5—Impedance plot of an inverted-V antenna cut for 29 MHz. At A, a plot of impedances, measured using the noise bridge, at the end of a 74-foot length of Columbia 1188 coaxial cable. At B, the actual antenna-impedance plot (found using the transmission-line equation to remove the effects of the transmission line).



impedance. If this effect is present, it can be minimized by making the measurements with a transmission line that is approximately an integer multiple of ½ λ long.

Conclusions

In this article, I've showed how to substantially increase the accuracy of an R-X noise bridge and how to use such a bridge to measure the characteristics of coaxial cable and to measure MF, HF and VHF antenna impedances. A number of other useful impedance measurements can be made with noise bridges, including component impedances. For instance, the improved noise-bridge I've presented here is sensitive enough to measure the inductance of a 1-inch length of wire at 30 MHz.

I hope that others will use this information to prepare better articles for *QST*. In particular, it would be valuable for antenna designs published in future issues of *QST* to include *measured* radiation-resistance and -reactance curves. The improved noise-bridge design discussed here makes that possible—at relatively low cost.

Notes

[1]B. Hale, ed, *The 1989 ARRL Handbook* (Newington: ARRL, 1988), pp 2-22 through 2-29.
[2]L. Wolfgang, ed, *The ARRL Extra Class License Manual,* 3rd ed (Newington: ARRL, 1988), Chapter 5.
[3]J. Belrose, "RX Noise Bridges," *QST*, May 1988, pp 34-35, 39.
[4]G. Hall, ed, *The ARRL Antenna Book,* 15th ed. (Newington: ARRL, 1988), Chapter 28.
[5]S. Ramo, J. Whinnery and T. Van Duzer, *Fields and Waves in Communication Electronics,* Jan 1967, Chapter 1.

APPENDIX

The impedance transformation that occurs when a signal propagates on a transmission line can be solved either graphically (using a Smith chart) or numerically, (using the transmission-line equation). The transmission line may be either open-wire line or coaxial cable. With the advent of personal computers, calculating impedance transformation in a transmission line is more easily and accurately done by numerically solving the transmission-line equation than by using the Smith chart. The impedance transformation along a transmission line is given by[5]

$$Z_i = Z_0 \left(\frac{Z_L \cosh(\gamma \ell) + Z_0 \sinh(\gamma \ell)}{Z_0 \cosh(\gamma \ell) + Z_L \sinh(\gamma \ell)} \right) \quad \text{(Eq 17)}$$

where
Z_i = input impedance of the transmission line
Z_0 = characteristic impedance of the transmission line
Z_L = load impedance at the end of the transmission line
ℓ = length of the transmission line
γ = complex propagation constant ($\gamma = \alpha + j\beta$)
α = attenuation constant in nepers per unit length (1 neper = 8.69 dB)
β = phase constant in radians per unit length

The impedances and the propagation constant may be complex numbers. The complex hyperbolic sine and cosine may be found by

$$\sinh(\alpha\ell + j\beta\ell) = \cos(\beta\ell)\sinh(\alpha\ell) + j\sin(\beta\ell)\cosh(\alpha\ell) \quad \text{(Eq 18)}$$

$$\cosh(\alpha\ell + j\beta\ell) = \cos(\beta\ell)\cosh(\alpha\ell) + j\sin(\beta\ell)\sinh(\alpha\ell) \quad \text{(Eq 19)}$$

$$\sinh(\alpha\ell) = \frac{e^{\alpha\ell} - e^{-\alpha\ell}}{2} \quad \text{(Eq 20)}$$

$$\cosh(\alpha\ell) = \frac{e^{\alpha\ell} + e^{-\alpha\ell}}{2} \quad \text{(Eq 21)}$$

For finding the load impedance (with a known transmission-line input impedance), the transmission-line equation is best written as

$$Z_L = Z_0 \left(\frac{Z_i \cosh(\gamma\ell) - Z_0 \sinh(\gamma\ell)}{Z_0 \cosh(\gamma\ell) - Z_i \sinh(\gamma\ell)} \right) \quad \text{(Eq 22)}$$

Most antenna measurements are made through a fixed length of coaxial cable. Therefore, we'll assume that $\alpha\ell$ is a single unit that we'll call the attenuation of the cable. This is commonly measured in decibels, but must be converted to nepers for use in the transmission-line equation. The phase constant can be expressed as a function of frequency and the length of the transmission line by:

$$\beta\ell = 2\pi \frac{f}{f_\lambda} \quad \text{(Eq 23)}$$

where
f = frequency of operation
f_λ = frequency at which the transmission line is 1 electrical λ long

A shorted transmission line is used to measure f_λ. To do this, find a frequency at which the transmission line has zero reactance and a low resistance (less than the characteristic resistance of the transmission line). We'll call this frequency f_n. Increase the frequency until the next zero-reactance, low-resistance point is found. We'll call this f_{n+2}. (The n indicates the number of quarter wavelengths that are present on the transmission line; n is always an integer.)

$$n = \frac{2 f_n}{f_{n+2} - f_n} \quad \text{(Eq 24)}$$

where n = 2, 4, 6,...

$$f_\lambda = \frac{4 f_n}{n} \quad \text{(Eq 25)}$$

The value of f_λ calculated in Eq 25 assumes that the transmission line has a nonreactive characteristic impedance. This is generally not true, but Eq 25 is accurate nonetheless; it yields an error of less than 2.5% for a transmission line with less than 3 dB loss and a reactive characteristic-impedance component of less than 10 Ω.

Transmission-line characteristic impedance is almost always complex. Good coaxial cable has a very small reactive characteristic-impedance component (on the order of a few ohms). Cable characteristic impedance is most easily calculated by placing a load at one end of the cable and measuring the impedance at the other end at two frequencies separated by $f/_\lambda/4$. The input impedance of the cable is then

$$Z_i(f) = Z_0 \left(\frac{Z_L \cosh(\gamma\ell) + Z_0 \sinh(\gamma\ell)}{Z_0 \cosh(\gamma\ell) + Z_L \sinh(\gamma\ell)} \right) \quad \text{(Eq 26)}$$

(continued on page 52)

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(continued from page 32)

$$Z_i(f+f_\lambda/4) = Z_0\left(\frac{Z_L\sinh(\gamma\ell) + Z_0\cosh(\gamma\ell)}{Z_0\sinh(\gamma\ell) + Z_L\cosh(\gamma\ell)}\right) \quad \text{(Eq 27)}$$

Eqs 26 and 27 can be manipulated so that the characteristic impedance can be found by

$$Z_0 = \sqrt{Z_i(f)\ Z_i(f+f_\lambda/4)} \quad \text{(Eq 28)}$$

The square root is complex, and may be calculated with a scientific calculator using the Eqs 29 through 33.

$$Z = R + jX = Z_i(f)Z_i(f + f_\lambda/4) \quad \text{(Eq 29)}$$

$$|Z| = \sqrt{R^2 + X^2} \quad \text{(Eq 30)}$$

$$R_0 = \sqrt{|Z|}\cos\left(½ \tan^{-1}\left[\frac{X}{R}\right]\right) \quad \text{(Eq 31)}$$

$$X_0 = \sqrt{|Z|}\sin\left(½ \tan^{-1}\left[\frac{X}{R}\right]\right) \quad \text{(Eq 32)}$$

$$Z_0 = R_0 + jX_0 \quad \text{(Eq 33)}$$

Transmission-line attenuation can be calculated after using this transmission-line impedance equation:

$$Z_i = Z_0\left(\frac{Z_L[\cos(\beta\ell) + j\alpha\ell\sin(\beta\ell)] + Z_0[\alpha\ell\cos(\beta\ell) + j\sin(\beta\ell)]}{Z_0[\cos(\beta\ell) + j\alpha\ell\sin(\beta\ell)] + Z_L[\alpha\ell\cos(\beta\ell) + j\sin(\beta\ell)]}\right) \quad \text{(Eq 34)}$$

This equation yields an error of less than 5%—as long as the transmission-line loss is less than 3 dB. If the transmission line is an odd multiple of a quarter wavelength (n = 1, 3, 5,...) and is terminated by an open circuit, or if the transmission line is an even multiple of a quarter wavelength (n = 2, 4, 6,...) and is terminated by a short circuit, the input impedance is given by

$$Z_i = \alpha\ell\, Z_0 \quad \text{(Eq 35)}$$

The attenuation of this transmission line can be found by

$$\alpha\ell = \frac{R_i}{R_0} \quad \text{(Eq 36)}$$

where R_i and R_0 are the resistive parts of the input impedance and the characteristic impedance, respectively. The transmission-line attenuation increases with frequency. An estimate for this attenuation is given by

$$\alpha\ell(f) = \alpha\ell(f_\alpha)\left(\frac{f}{f_\alpha}\right)^\sigma \quad \text{(Eq 37)}$$

where
$0.5 < \sigma < 1$

This equation can be used to interpolate between unmeasured values of attenuation. For most coaxial cables, $\sigma = 0.5$ works well.